Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE PRINCIPAL AXES OF STELLAR MOTION

By H. Raymond

DUDLEY OBSERVATORY, ALBANY, N. Y.

Communicated by E. B. Frost, May 28, 1917

Suppose the stars of known motion transferred to a fixed point O, and allowed to move with their present velocities for a definite length of time. They will have expanded into a cluster, the 'velocity-figure.' According to the older idea that the stars were moving 'at random,' this cluster should be spherical. Since J. C. Kapteyn showed[1] the figure to be elongated, several methods of finding the amount and direction of elongation have been proposed and used. Some of these make use only of the directions of observed proper-motions, leaving the amounts of those motions wholly out of account, thus avoiding the difficulties caused by our ignorance of the distances of most stars; and are based on some hypothesis as to the actual shape of the velocity-figure. The most notable of these are A. S. Eddington's[2] and K. Schwarzschild's.[3] The method here used makes no assumption as to the form of the velocity-figure, and is essentially a process of finding its principal momental axes. It thus belongs to the same *genus* as those used by C. V. L. Charlier,[4] K. W. Gyllenberg,[5] A. S. Eddington and W. E. Hartley,[6] and H. C. Plummer.[7]

Let the coordinates of a star in the velocity-figure, referred to suitable axes through O,—say to the First Point of Aries, to (6h, 0°), and to the North Pole, respectively,—be x', y', z'; the direction cosines of an arbitrarily chosen line be l, m, n; and the projection of the star S upon this line be P. Then the component of motion in the direction l, m, n, is $p = OP = lx' + my' + nz'$. Representing the mean of a quantity by enclosing it in square brackets,

$$[pp] = l^2[x'x'] + m^2[y'y'] + n^2[z'z'] + 2\,lm\,[x'y'] + 2\,ln\,[x'z'] + 2\,mn\,[y'z'] \quad \text{(A)}$$

The axis of preference is defined as the line for which $[pp]$ is a maximum; for the axis of avoidance $[pp]$ is a minimum.

Newcomb, in his paper 'on the Principal Planes toward which the Stars Tend to Crowd,' applies to another problem an essentially similar process, and gives a detailed solution by Lagrange's method for a maximum or minimum of (A), from which the procedure of this investigation is adapted. Putting $A = [x'x']$, $B = [y'y']$, $C = [z'z']$, $D = [x'y']$, $E = [x'z']$, $F = [y'z']$, the maximum, minimax, and minimum values of $[pp]$ are the three roots of the cubic

$$\begin{vmatrix} (A-\lambda) & D & E \\ D & (B-\lambda) & F \\ E & F & (C-\lambda) \end{vmatrix} = 0 \qquad \text{(B)}$$

and the corresponding values of l, m, n, are derived by inserting these roots successively in the set of equations

$$\begin{aligned} (A-\lambda)l + \quad Dm + \quad En &= 0 \\ Dl + (B-\lambda)m + \quad Fn &= 0 \\ El + \quad Fm + (C-\lambda)n &= 0 \end{aligned} \qquad \text{(C)}$$

The coordinates x', y', z' are referred to the unknown center of gravity of the velocity-figure. We can observe only the velocities x, y, z, relative to the moving sun. But we readily find that

$$\begin{aligned} [x'x'] &= [xx] - [x]\cdot[x] \qquad & [x'y'] &= [xy] - [x]\cdot[y] \\ [y'y'] &= [yy] - [y]\cdot[y] & [x'z'] &= [xz] - [x]\cdot[z] \\ [z'z'] &= [zz] - [z]\cdot[z] & [y'z'] &= [yz] - [y]\cdot[z] \end{aligned} \qquad \text{(D)}$$

The quantities $[x]$, $[y]$, $[z]$, are the negatives of the coordinates of solar motion, according to the method of Bravais.

We are limited to the use of the proper-motions alone, or the radial velocities alone, since to combine them requires a knowledge of individual parallaxes which we do not possess. The rectangular components of the proper-motion part (tangential to the celestial sphere) of the stellar motions, resolved as before, may be expressed in terms of the proper-motions μ, μ', and the spherical coordinates of the stars, while μ, μ' may be expressed in terms of x, y, z. Designating the tangential part by subscript l, we readily obtain

$$x_1 = x(1-\cos^2\alpha\cos^2\delta) - y\sin\alpha\cos\alpha\cos^2\delta - z\cos\alpha\sin\alpha\cos\delta,$$

and similarly for y_1 and z_1. From these x, x_1, etc., are found by multiplication. The means of these quantities are found by integration over the sphere. They are, cleared of fractions,

$$\begin{aligned} 3[x_1] &= 2[x] \qquad & 15[x_1y_1] &= 7[xy] \\ 3[y_1] &= 2[y] & 15[x_1z_1] &= 7[xz] \\ 3[z_1] &= 2[z] & 15[y_1z_1] &= 7[yz] \\ 15[x_1x_1] &= 8[xx] + [yy] + [zz] \\ 15[y_1y_1] &= [xx] + 8[yy] + [zz] \\ 15[z_1z_1] &= [xx] + [yy] + 8[zz] \end{aligned} \qquad \text{(E)}$$

The admissibility of these integrations depends upon certain conditions, which may be stated in the form of three hypotheses:

I. The stars are evenly distributed over the sky.

II. The distribution in distance is the same in different parts of the sky.

III. The motions of the stars are distributed in the same way in all accessible parts of space.

These hypotheses are of course not rigorously true. III is open to objection, but if it is not at least approximately true we have no general problem, merely a number of local problems. After the mean square velocities in the three principal directions are known, failure of II, so far as effect of different mean square parallax in different parts of the sky is concerned, may be taken care of and a new approximation made. The theoretical value of $[x_1x_1] + [y_1y_1] + [z_1z_1]$ can be computed for each region of the sky and compared with the observed values. A difference may be considered as due to the stars being nearer or farther than the average and factors applied accordingly. These approximations converge very rapidly; in fact, the error from the source considered is small, even in the first solution. Errors due to the spread or range of the stars in distance are not corrected by this process, but must usually be negligible. Hypothesis I can be made valid by simply weighting the material proportionally to the area of the sky covered, instead of proportionally to the number of stars.

The material used was the 5943 proper-motions less than 80″ per century from Boss's Preliminary General Catalogue. Of these, 5384 were less than 20″ per century (allowing for solar motion[8]) and were called "S"; the remaining 559 stars of larger proper-motion were called "L." Group S was subdivided according to type of spectrum as follows: B, types Oe5 to B 5, 492 stars; A, B 8 to A 4, 1647; F, A 5 through F, 656; G, 446 stars; K, 1227; M, 223; X, including stars of unknown type—mostly faint—and some of types N, O, and P, 693 stars.

The numerical results appear in the accompanying tables. Table 1 gives, in column 1, the designation of the group; in 2 and 3 the Right Ascension and Declination respectively of the direction in which $[pp]$ is a maximum—the vertex of preferential motion; in 4 and 5, the same for direction of least motion. The third direction, in which $[pp]$ is a minimax, is necessarily at right angles to each of these, hence is not given. The last three columns give the value of $[pp]$ for these three directions. Unit of motion, 1″ per century.

Table 2 gives the solar motion derived at the same time. *A* and *D* are the Right Ascension and Declination of the Apex, *M* the amount of solar motion. The last three columns are given as a means of comparing the various groups. If the means without regard to sign of the components of motion in the three principal directions be $[u]$, $[v]$, $[w]$, we have

$$[u] = \sqrt{(2\lambda_1/\pi)},\ [v] = \sqrt{(2\lambda_2/\pi)},\ [w] = \sqrt{(2\lambda_3/\pi)}$$

provided the components are distributed according to the error law. They have merely the broad characters of such distribution,—sufficient to justify the process in the same sense that the customary method of finding the 'probable error,' without inquiring too diligently as to whether the errors of observation are actually distributed in the manner assumed in theory, is justified.

TABLE 1

SYSTEMATIC MOTION

GROUP	POLE OF PREFERENCE		POLE OF AVOIDANCE		MEAN SQUARE MOTIONS		
	A_1	D_1	A_3	D_3	λ_1	λ_2	λ_3
B............	100°6	−38°4	180°4	+12°6	4.58	1.94	1.33
A............	92.4	+1.8	182.9	+15.2	25.65	9.15	2.57
F............	93.1	+2.1	185.2	+45.9	58.05	28.78	22.66
G............	93.8	+3.4	185.4	+24.4	32.26	15.81	9.96
K............	93.1	+11.3	191.2	+35.3	29.30	18.71	11.46
M............	94.0	+25.3	205.1	+37.2	28.14	12.58	3.48
X............	83.9	+30.2	192.6	+29.0	21.08	12.72	5.33
S............	91.9	+ 4.2	183.8	+24.9	28.91	14.99	9.89
L............	92.5	+35.3	211.7	+34.5	745.6	261.4	194.7
All...........	92.3	+16.9	184.7	+ 8.1	107.3	58.7	31.8

TABLE 2

SOLAR MOTION

GROUP	NUMBER	SOLAR APEX			MEAN MOTIONS		
		A	D	M	[u]/M	[v]/M	[w]/M
B............	492	274°2	+39°3	2″33	0.73	0.47	0.40
A............	1647	266.7	+27.9	4.03	1.00	0.60	0.32
F............	656	262.9	+27.0	6.36	0.96	0.67	0.60
G............	446	256.6	+45.8	2.60	1.74	1.22	0.97
K............	1227	272.5	+38.4	4.02	1.08	0.86	0.67
M............	223	272.2	+45.7	3.45	1.23	0.82	0.43
X............	693	271.4	+42.1	3.06	1.20	0.93	0.60
S............	5384	269.0	+32.1	3.83	1.12	0.81	0.66
L............	559	269.3	+31.5	22.17	1.01	0.61	0.52
All...........	5943	269.1	+32.3	5.42	1.53	1.13	0.83

In regard to these results the following points seem especially worth noting:

1. All groups show markedly unequal motions in three principal directions. In all groups the direction of greatest motion, or axis of preference, lies near the plane of the galaxy not far from its intersection with the equator, the axis of avoidance is nearly perpendicular to the galaxy.

Using radial velocities Gyllenberg[5] finds much the same axis of preference, but the other two axes practically interchanged. So do Eddington and Hartley[6] for later types, but their results for early types agree better with table 1. Are these differences real, in the sense of being typical of what may be expected whenever radial velocities and proper-motions are compared? If so, what peculiarity of distribution or of motions in space, out of the many that might easily be suggested, lies back of it? Here is a subject well worth investigation.

2. The separation of the poles of preference into two groups corresponding respectively to 'early' and 'late' types,[9] is confirmed. There seems to be some evidence of progression of the same character within each of the two groups of types, as if with advance of type the 6-hour vertex moved northward. With advance of type there is an increase of stellar as compared with solar motion; that in the preferred direction, however, increasing less than the other two components.

3. The apices of solar motion show the division into two groups already found by several investigators; A and F having apices in smaller R. A. and Decl. than K, M, and X. B seems to resemble the latter group. G is anomalous.

These results agree rather closely with those found by L. Boss by Airy's method,[10] and by the writer by Schwarzschild's method,[9] all three being based upon practically the same material. This agreement will be found to extend even to details.

4. The distribution of stars of type B is so peculiar that to weight the material according to area covered would have been unfair. The integrations which give equations (E) can be made over zones parallel to the galaxy, and the resulting relations used to correct each zone for perspective separately, the material then being combined with weights proportional to the number of stars. The results in the tables were thus found; a check solution by the ordinary method gave for the vertex (122°,-36°).

The vertex of preferential motion is near the antapex of solar motion. This may in part be a manifestation of the tendency, noted above, for the earlier types to have more southerly vertices. In part it is due to the fact that the stars' 'spread' in distance tends to produce a false elongation of the velocity-figure in the direction apex-antapex. This tendency is stronger the greater the spread, the smaller the velocity-figure relative to solar motion, and the more nearly spherical the figure. The last two causes, and possibly the first, operate strongly in the B stars. There is no appearance of such an effect in the other groups.

5. The separation of groups "S" and "L" was made some years ago, for a different purpose, and according to a method which made allowance for solar motion relative to group S, but not enough for L, and made none for preferential motion. Consequently, while S no doubt represents fairly well the smaller motions, L does not truly represent the large ones. In group "All," while the large motion characters are diluted by large numbers of small motion stars, the former have individually much more weight than the latter; this therefore is probably more representative that L of the large proper-motion stars. A re-division into small and large proper-motions, on principles better adapted to the purpose of this investigation, is in progress.

[1] *London, Rep. Brit Ass.*, 1905, (257).

[2] *London Mon. Not. R. Astr. Soc.*, **67,** 1906, (34).

[3] *Göttingen, Nachr., Math.-Phys. Kl.*, 1907, (614); 1908 (191).

[4] *Medd. Lunds Astr. Obs.*, (Ser. 2), No. 9, 1913.

[5] *Ibid.*, No. 59, and (Ser. 2), No. 13, 1914–15.

[6] *London Mon. Not. R. Astr. Soc.*, **75,** 1915, (521).

[7] *Ibid.*, **76,** 1915, (121).

[8] *Astr. J. Boston*, **612,** 1910, (98).

[9] *Ibid.*, **676,** 1915.

[10] *Ibid.*, **614, 623–4,** 1910–11.